$$\therefore p=1-\frac{4}{5\pi r^3}\left[\int_{\frac{1}{2}r}^{r}\int_0^{\theta}[15(\tfrac{1}{2}\pi-\theta)\cos\theta-10\cos\theta+10\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta-\tfrac{5}{2}\tan\tfrac{1}{2}\theta\sec^2\tfrac{1}{2}\theta]\times\right.$$

$$\left.x^2dxd\theta+\int_0^{\frac{1}{2}r}\int_0^{\frac{1}{2}\pi}[15(\tfrac{1}{2}\pi-\theta)\cos\theta-10\cos\theta+10\sin\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta-\tfrac{5}{2}\tan\tfrac{1}{2}\theta\sec^2\tfrac{1}{2}\theta]x^2dxd\theta\right]$$

$$=1-\frac{4}{5\pi r^3}\left[\int_{\frac{1}{2}r}^{r}\left(15(r-x)\cos^{-1}(\frac{r-x}{x})-10r-20\surd(2rx-r^2)\right.\right.$$

$$\left.\left.+\tfrac{65}{2}x-\frac{5x^2}{x+\surd(2rx-r^2)}\right)xdx+\tfrac{15}{2}\int_0^{\frac{1}{2}r}x^2dx\right]=\frac{4}{\pi}(8\log2-5).$$

Also solved with same result by the *PROPOSER*.

MISCELLANEOUS.

107. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

The index of refraction of a medium varying inversely as the square root of the distance, prove that the path of a ray of light in the medium is a cycloid.

Solution by the PROPOSER.

Taking the axis of y in the given plane and that of x at right angles to the y axis, letting $\mu=k/\surd x$ be the index of refraction, and $p=dy/dx$, we have, by the usual theory, for the differential equation to the path

$$\frac{dp/dx}{1+p^2}=\frac{1}{\mu}\left[\frac{d\mu}{dy}-\frac{d\mu}{dx}\frac{dy}{dx}\right]....(1).\quad \text{We have } \frac{d\mu}{dx}=-\frac{k}{2x^{\frac{3}{2}}}....(2),$$

and (1) becomes $\dfrac{dp/dx}{1+p^2}=\dfrac{p}{2x}$, or $\dfrac{dp}{p(1+p^2)}=\dfrac{dx}{2x}....(3)$.

Integrating, $\log\dfrac{p}{\surd(1+p^2)}=\log\surd x+C....(4)$.

Let $p=b$, when $x=a$; then $C=\log\dfrac{b}{a\surd(1+b^2)}$,

and (4) becomes $p=\dfrac{dy}{dx}=\dfrac{xdx}{\surd[(a^2/b^2)(1+b^2)x-x^2]}....(5)$, the differential equation to a cycloid.

Also solved by *G. B. M. ZERR*, and *L. C. WALKER*.

108. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

To divide the arc of a cardioid into eight equal parts.

Solution by L. C. WALKER, A.M., Professor of Mathematics, Petaluma High School, Petaluma, Cal.

Equation of the cordioid is $r=a(1+\cos\theta)$, and its complete perimeter is

$$S=2\int_0^\pi \sqrt{[a^2(1+\cos\theta)^2+a^2\sin^2\theta]}d\theta=4a\int_0^\pi \cos\tfrac{1}{2}\theta d\theta=8a,$$

the arc S being measured from the point where the curve crosses the initial line.

Let θ' be the point of the first division; then

$$2a\int_0^{\theta'}\cos\tfrac{1}{2}\theta d\theta=4a\sin\tfrac{1}{2}\theta'=a;$$

from which $\theta_{1,\,7}=\pm2\sin^{-1}\frac{1}{4}$. Similarly, $\theta_{2,\,6}=\pm2\sin^{-1}\frac{1}{2}$, $\theta_{3,\,5}=\pm2\sin^{-1}\frac{3}{4}$, $\theta_4=\pi$, and $\theta_8=0$.

Excellent solutions were received from *G. B. M. ZERR*, and *F. P. MATZ*.

109. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

Find the latitude of the place where the sun's center remains above the horizon for a hundred successive days.

Solution by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy, Defiance College, Defiance, O.

I. The solar phenomenon in question will begin May 2 and end August 10. For May 2 at Washington mean noon, according to the Director of United States Nautical Almanac Office, the declination of the sun is $\delta=+15°\ 18'\ 54.7''$. Representing the hour-angle of the sun by h, we have $\cos h=-\tan\lambda\tan\delta$....(1).

Since there is to be no setting of the sun, we may put $\cos h=-1$.

$\therefore\ \lambda=\tan^{-1}\left(\dfrac{1}{\tan\delta}\right)=\tan^{-1}(\cot\delta)=90°-\delta,=74°\ 41'\ 5.3''$, which is the terrestrial latitude of the place required. This solar phenomenon may be observed on Melville Island.

II. The celestial longitude of the sun for mean noon Washington can be determined in various ways to be about $\psi=41°\ 34'\ 25''$; the obliquity of the ecliptic may be taken $\omega=23°\ 27'\ 8''$; then $\lambda=90-\delta=90-\sin^{-1}(\sin\psi\times\sin\omega),=$ $74°\ 43'\ 16''$.

Also solved by *G. B. M. ZERR*, and *H. C. WHITAKER*.

110. Proposed by E. W. MORRELL, South Trowbridge, Vt.

If a and b be the sides of a triangle, A and B the angles opposite, then will $\log b-\log a=\cos3A-\cos2B+\frac{1}{2}(\cos4A-\cos4B)+\frac{1}{3}(\cos6A-\cos6B)+....$

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

$1+\cos2A+\frac{1}{2}\cos4A+\frac{1}{3}\cos6A+....=-\frac{1}{2}\log(2-2\cos2A)=-\log(2\sin A)$.

$1+\cos2B+\frac{1}{2}\cos4B+\frac{1}{3}\cos6B+....=-\log(2\sin B)$.

See Trigonometry for the summation of these series.

$\therefore\ \cos2A-\cos2B+\frac{1}{2}(\cos4A-\cos4B)+....=\log(\sin B/\sin A)=\log(b/a)$.

Also solved by *F. P. MATZ*, and *J. SCHEFFER*.